Reg. No. | | | | | | | | | | | |

Question Paper Code : 11294

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

Civil Engineering

MA 1101 – MATHEMATICS – I

(Common to all branches)

(Regulations 2008)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Find the sum and product of the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$.

2. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

3. Show that the sphere with center (1, 2, –2) and radius 3, passes through the origin.

4. Write down the equation of the cylinder whose axis is y – axis and the distance between the axis and the generating curve is a.

5. Find the radius of curvature of the parabola $y^2 = 4ax$ at $y = 2a$.

6. If the center curvature of a curve at a variable point 't' on it is $(2a + 3at^2, -2at^3)$, find the evolute of the curve.

7. If $x^y + y^x = c$, find $\frac{dy}{dx}$.

8. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

9. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$.

10. Convert the equation $xy'' - 3y' + x^{-1} y = x^2$ as a linear equation with constant coefficients.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Using Cayley Hamilton theorem, find A^{-1} when **(8)**

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

(ii) Using similarity transformation diagonalize the matrix. **(8)**

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

(b) (i) Reduce the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy + 6yz - 10zx$ to canonical form through an orthogonal reduction. **(8)**

(ii) Find the eigen values and eigen vector of the matrix. **(8)**

$$A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

12. (a) (i) Show that the lines $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z-3}{4}$ and $\frac{x-1}{4}=\frac{y-1}{3}=\frac{z-1}{5}$ are coplanar. Find the equation to the plane containing them. **(8)**

(ii) Find the centre and radius of the circle $x^2+y^2+z^2-2x-2y-4z-10=0$, $x+y+2z=8$. **(8)**

OR

(b) (i) Find the length and equation of the line of shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x-3}{3}=\frac{y+7}{2}=\frac{z-6}{4}$ **(8)**

(ii) Find the equation of the cone whose vertex is the point (1, 2, 3) and whose guiding curve is the circle $x^2+y^2+z^2=4$, $x+y+z=1$. **(8)**

13. (a) (i) Find the radius of the curvature for the curve $y^2=12x$ at (3, 6). **(8)**

(ii) Find the equation of the envelope for the family of the lines $\frac{x}{a}+\frac{y}{b}=1$ with the condition on the parameters a + b = c for a constant C. **(8)**

OR

(b) (i) Find the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at a point (a, a). **(8)**

(ii) Find the evolute of the parabola $y^2=4ax$, considering it as the envelope of its normals. **(8)**

14. (a) (i) Find and classify the extreme values, if any, of the function $f(x, y)=x^2+y^2+xy+\frac{1}{x}+\frac{1}{y}$. **(8)**

(ii) Find the Tailor's series expansion of $e^x \sin y$ near the point $(-1, \pi/4)$, upto the third degree term. **(8)**

OR

(b) (i) If $x = r \cos \theta$, $y = r \sin \theta$, prove that the equation $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ is equivalent to $\frac{\partial u}{\partial r} + \frac{1}{r} \tan\left(\frac{\pi}{4} - \theta\right) \frac{\partial u}{\partial \theta} = 0$. **(8)**

(ii) Find the maximum value of $x^m y^n z^p$, when $x + y + z = a$, using Lagrange multiplier method. **(8)**

15. (a) (i) Solve : $(D^4 - 2D^3 + D^2)\, y = x^2 + e^x$. **(8)**

(ii) Solve : $(x^2D^2 - xD + 1)\, y = \left(\frac{\log x}{x}\right)^2$ **(8)**

OR

(b) (i) Solve the simultaneous equations : **(8)**

$$\frac{dx}{dt} + 2x - 3y = 5t \; ; \; \frac{dy}{dt} - 3x + 2y = 2e^{2t}$$

(ii) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = x \sin x$. **(8)**
